SQUENCE AND SERIES

Part I

WHAT IS A PROGRESSION ?

A progression is a list of things (usually numbers) that are in order.

Example : 2 (1st term), 4 (2nd term), 8..... (3rd term) ← Dots Denote Infinite Progression

TYPE OF PROGRESSION

- Arithmetic Progression
- Geometric progression
- Harmonic Progression
- Arithmetico Geometric progression
- Miscellaneous Progression

Arithmetic Progression

Definition

A pattern of numbers that increases or decreases by a constant number.
E.g. 4, 7, 10, 13.....

General Progression

General form of an arithmetic progression is given as $a, a+d, a+2d..., a+(n-1)d$

Where: a – First term ; d – Common difference

n^{th} term

General term of an arithmetic progression is given as $T_n = a + (n-1)d$

Sum of 'n' terms

If 'n' terms $a, a+d, a+2d..., a+(n-1)d$ are in arithmetic progression Then the sum of 'n' terms:

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

Airthmetic Mean

If $a_1, a_2, ..., a_n$ are in arithmetic progression then the Arithmetic Mean (AM) is:

$$A_m = \frac{a_1 + a_2 + ... + a_n}{n} \text{ or } = \frac{S_n}{n}$$

If $A_1, A_2 ..... A_n$ are 'n' arithmetic means between two numbers 'a' and 'b' then $a, A_1, A_2 ..... A_n, b$ are in AP.

Where common diffrence $d = \frac{b-a}{n+1}$ and arithmetic means are $A_i = a + i\frac{b-a}{n+1}$

Geometric Progression

Definition

The progression, where the ratio of successive terms of a progression is constant
E.g. 4, 8, 16, 32, 64, ... here the common ratio is 2.

General Progression

General form of a geometric progression is given as $a, ar, ar^2, \ldots. ar^{n-1}$

Where: a – First term | r – Common ratio

n^{th} term

General term of a geometric progression is given as $T_n = a \cdot r^{(n-1)}$

Sum of 'n' terms

If 'n' terms $a, ar, ar^2, \ldots. ar^{n-1}$ are in geometric progression then the sum of 'n' terms:

$$S_n = \frac{a(r^n-1)}{r-1}; \; r \neq 1$$

Geometric Mean

If $a_1, a_2, \ldots, a_n$ are in geometric progression then the geometric mean (GM) is:

$$G_m = (a_1 \cdot a_2 \cdot a_3 \ldots. a_n)^{1/n}$$

If $G_1, G_2 \ldots. G_n$ are 'n' geometric means between two numbers 'a' and 'b' then $a, G_1, G_2 \ldots. G_n, b$ are in G.P.

Where common ratio $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$ and geometric means are $G_i = a\left(\frac{b}{a}\right)^{\frac{i}{n+1}}$

Arithmetico Geometric Progression

Definition

The result of the multiplication of a geometric progression with the corresponding terms of an arithmetic progression

General Progression

$a, (a+d)r, (a+2d)r^2, (a+3d)r^3 \ldots$ Where:

a – First term | r – Common ratio of GP | d – Common difference of AP

n^{th} Term

General term of a arithmetico geometric progression is $T_n = [a+(n-1)d]r^{(n-1)}$

Sum of 'n' Terms

$a, (a+d)r, (a+2d)r^2, \ldots.$ are in AGP then sum of the terms is:

$$S_n = \frac{a}{1-r} + \frac{rd(1-r^{n-1})}{(1-r)^2} - \frac{[a+(n-1)d]r^n}{1-r}$$

If $|r|<1$ and 'n' tends to infinity then sum of infinite terms is:

$$\lim_{n\to\infty} S_n = \frac{a}{(1-r)} + \frac{rd}{(1-r)^2}$$